Reflections

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The Most Important Math for Learning Physical Science

Part II – Proportionality

Bob Stair

(Aside from basic computational skills, there are two important bits of mathematics—histograms and proportionality—that are needed to get the most out of an inquiry-based physical science course such as Introductory Physical Science. *The first article in this series addressed histograms. This article emphasizes the importance of proportionality as we attempt to model nature through formulas.)*

In science, students are presented with formulas, such as

$$\text{Distance} = \text{Speed} \times \text{Time}$$

or

$$\text{Mass} = \text{Density} \times \text{Volume}\ .$$

Teachers then often give students worksheets filled with problems that drill the formulas, and most students soon master the algorithm for using each formula, but does this practice really give students an understanding of what the formula really means as a description of nature? *Where do formulas come from? What do they tell us about the world around us?*

Formulas are not statements that authors dream up to challenge students—as I am sure some students think! As with everything in *Introductory Physical Science* (*IPS*), the "discovery" of a formula results from the use of both scientific instruments and the tools of mathematics to collect and carefully analyze real data. The formula then provides us with a model for how nature behaves by describing what we observe in nature.

As *IPS* students "question nature," they collect data using various scientific instruments. The choice of instruments depends on the objectives of each specific investigation, but some instruments—such as balances, thermometers, and spring scales—are used repeatedly in many experiments. Similarly, there are certain mathematical tools that are used repeatedly. For example, one of the most important mathematical ideas for beginning physical science students is the concept of proportionality—the idea that doubling one quantity leads to a doubling of a second quantity, tripling the first leads to a tripling of the second, and so on. "Questioning nature" very often translates to "Which quantities are proportional to each other?"

The search for proportionality leads to each of the formulas presented at the beginning of this

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article. For example, suppose that a car is traveling at a constant speed. By noting the odometer reading every minute, we might collect the data shown in Table 1.

Table 1 Simulated Data for a Constant Speed Car

Time (min)	Total Distance Traveled (km)
0.0	0.0
1.0	1.4
2.0	2.8
3.0	4.2
4.0	5.6
5.0	7.0

To analyze this data, we can construct a graph, such as the one shown below in Figure 1. There are two features to be noted about this graph:

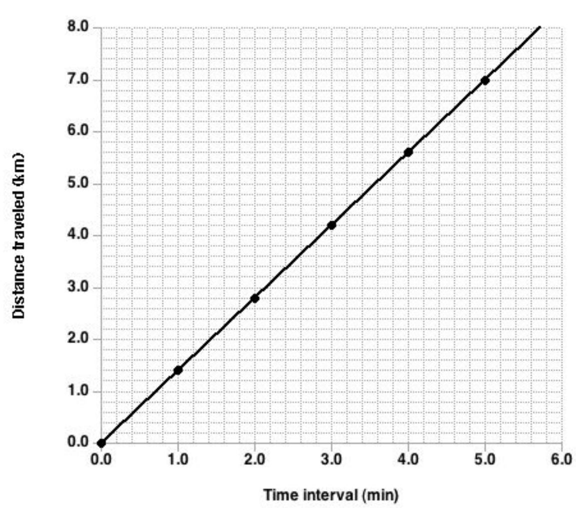
1. The graph is a straight line.
2. The line passes through the origin (0,0).

Whenever a graph has these characteristics, we can say that the quantity on the vertical axis is proportional to the quantity on the horizontal axis.

Distance traveled is proportional to time interval.

You can check this relation by selecting points on the graph that represent a doubling or a tripling of the time interval—for example, the points where t=2 minutes and t=4 minutes, or t=1 minute and t=3 minutes. Does the distance traveled double when the time interval is doubled? Does it triple when the time interval is tripled?

Figure 1 A Graph of the Simulated Data Presented in Table 1



Of course, we might have been able to deduce that the distance traveled is proportional to the time interval by asking the same questions about the simulated data presented in Table 1. It is a rare student, however, who will be able to recognize proportionality by simply looking at raw data with all of its variations due to the sensitivities of measuring instruments. By smoothing out the variations, a graph allows us to see the overall relationship more easily.

OK…we have found that the distance traveled is proportional to the time interval for a car traveling at a constant speed. How does that proportionality become the formula relating distance to speed and time? To achieve that transition, we make use of the relationship between proportions and equalities…*any proportion can be converted to an equality by introducing a proportionality constant having the appropriate units.*

If

Distance is proportional to time interval

then

$$\text{Distance} = (\text{proportionality constant}) \times \text{time} .$$

(Notice that this equation is a "real world" example of something students have already seen in math class. It is of the same form as $y=mx$, the equation of a line that passes through the origin.)

How do we determine the value of the proportionality constant? Since the distance vs. time graph shown in Figure 1 passes through the origin, we can pick any point on the line to calculate the constant's value. For example, at 4.4 minutes, the car has traveled about 6.2 km. Substituting these values into the above equation yields

$$6.2\text{ km} = (\text{proportionality constant}) \times 4.4\text{ min} .$$

By solving this equation, we can see that the value of the proportionality constant is 1.4 km/min for the simulated data in Table 1.

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Note that the proportionality constant has units of km/min. This is of the same form as meters per second, miles per hour, and kilometers per hour. Apparently, the proportionality constant has the same units as speed, and indeed, it is the speed of the car! So we have determined the formula relating distance, speed, and time!

$$\text{Distance} = \text{Speed} \times \text{Time}$$

A similar analysis of mass and volume data for a given substance yields

Mass is proportional to volume

so

$$\text{Mass} = (\text{proportionality constant}) \times \text{volume} .$$

In this case, the proportionality constant may have units such as kilograms per cubic meter or grams per cubic centimeter. It is density!

$$\text{Mass} = \text{Density} \times \text{Volume}$$

Sometimes a quantity is determined to be proportional to two or more other quantities. In such a case, we must utilize a property of proportions. *If one quantity is proportional to two or more other quantities, it is proportional to the product of those quantities.* So if

Quantity A is proportional to Quantity B

and

Quantity A is proportional to Quantity C

then

Quantity A is proportional to (Quantity B × Quantity C) .

Using an example from *IPS*, students find that a change in gravitational potential energy (GPE) is proportional to the mass of an object.

Change in GPE is proportional to mass.

They also see that the change in GPE is proportional to the vertical distance that the object rises or falls.

Change in GPE is proportional to distance.

According to the property of proportions cited above, this means

Change in GPE is proportional to (mass × distance) .

Consequently,

$$\text{Change in GPE} = (\text{constant}) \times \text{mass} \times \text{distance} .$$

Using the techniques described here, one can better understand the origins of each of the formulas listed in Table 2.

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Table 2 How Formulas Used in IPS Result From Proportionalities

Related Quantities	Proportionalities	Proportionality Constant		Formula
		Symbol	Name	
Distance, time interval	Distance traveled at a constant speed is proportional to time interval.	v	Speed	Distance = speed × time
Mass, volume	Mass is proportional to volume.	D	Density	Mass = density × volume
Weight, mass	Weight is proportional to mass.	g	*	Weight = g × mass
Elastic force, distance spring is stretched or compressed	Elastic force is proportional to the distance a spring is stretched or compressed.	k	Spring constant	Force = (spring constant) × (distance stretched)
Change in GPE, mass, vertical distance	Change in GPE is proportional to both the mass and the distance an object rises or falls.	g	**	Change in GPE = g × mass × (vertical distance)
Change in thermal energy (ThE), mass, change in temperature	Change in ThE is proportional to both mass and change in temperature.	c	Specific heat	Change in ThE = (specific heat) × mass × (change in temperature)
Change in velocity, mass, force, time interval	Change in velocity is proportional to the force exerted on an object and the time interval that the force is applied, and it is inversely proportional to the mass of the object.	***	***	Change in velocity = $\frac{\text{force} \times \text{(time interval)}}{\text{mass}}$
Frictional force, weight	Frictional force experienced by and object is proportional to its weight.	μ****	Coefficient of friction	Force = (coefficient of friction) × weight

* In *IPS*, the word "acceleration" is not introduced, both because it is not needed and because it is unrealistic to expect beginning physical science students to grasp its physical meaning. Therefore, in keeping with our approach of not introducing vocabulary until there is a need for it, this constant remains unnamed in *IPS*.

** This is the same constant as the one in the proportionality between weight and mass, albeit with different units (J/kg•m vs. N/kg).

*** If change in velocity is measured in meters/second, mass in kilograms, force in N, and time in seconds, the proportionality constant has a value of 1 (unitless). As a result, it does not warrant a symbol or name, and it is omitted from the formula.

**** The Greek letter mu (μ) is used to represent the coefficient of friction.

In *IPS*, this "data analysis-to-formula" process is used repeatedly, as shown in Table 2. In fact, it is essential to the approach taken when introducing Newton's Second Law in Chapter 16, Force and Motion in a Straight Line. With this data-driven approach, students learn more than just an algorithm for solving problems or a formula that they may remember and be able to repeat years later—but without understanding. They come to understand what a formula tells us about how nature behaves…and it all starts with proportionality!

(In the next Reflections…what if a data graph does not show a straight line through the origin?